MAGNITUDE RISK ANALYSIS IN LIBYA

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ABSTRACT

An attempt to estimate the maximum magnitude of earthquake in Libya (21° – 34° N), (8° – 24.5° E) by using the extreme value theory was presented here. The gathered earthquakes data which have occurred in and around Libya in the period of 1932 – 2000 are used to investigate and determine the recurrence relationship of the Gumble type (I) distribution, using the least square technique for estimating the statistical parameters.

The maximum magnitude occurrence predictability was tested and given by predicting the magnitudes corresponding to the return period as well as the probability of the occurrence of earthquake risk over various design period.

INTRODUCTION

The principal engineering use of earthquake data, both macro-seismic and instrumentation, is to establish in a given geographic region the recurrence laws as well as to define and assess acceptable seismic risk levels for the design of different types of engineering structures. According to Ambraseys (1978) the estimation of the maximum magnitude earthquake and its occurrence in space and time is the most difficult of all seismic risk parameters to assess. Among the different methods employed in estimating the largest possible earthquake magnitude, that which is based on the theory of extremes is in frequent use (Karnik and Algermissen, 1978).

Extreme value statistics developed by Gumble (1958) provide a convenient method to obtain estimates of the frequencies of occurrence of natural events (e.g. meteor-
ological, hydrological, geological conditions …etc.) on the extreme of statistical
distribution and to estimate recurrence times for these events.

The application of Gumbel’s theory to earthquake magnitude data dates back to the
early work of Nordquist (1845).

The significance of this study induced later investigations along similar lines which
covered various regions of the world over different time samples (see, e.g. Shakal &
Willis 1972, Yeqlalaplp and Kuo, 1974; Burton, Main and Long, 1983).

In the present paper we use Gumbel type (I) asymptotic distribution parameters in
estimating maximum magnitude and return period for earthquakes occurrence in Libya
and it’s bordering region over 68yr period.

Although neither the concepts nor the techniques used in this work are new, the
results arrived at nevertheless provide quantitative basis for evaluating moderate levels
of earthquake risk.

PREVIOUS WORK

However, the only two investigations into the seismicity of the country of Libya are
given by Mallick, 1977 and Kebeasy 1980. While the distribution of earthquakes in
space and time suggest classification of Libya into three major active seismic zones and
an aseismic one imposed by Mallick, 1977 and Kebeasy, 1980, on the other hand,
investigated the frequency of the earthquake occurrence even some seismic engineering
aspects. In his investigation the frequency of occurrence of shallow earthquakes which
have been occurred in Libya over the past 40 years are considered and then the
relationship between logN and the corresponding magnitudes for actually observed
earthquakes was given as well as (a and b) values were determined using least square
solutions.

Actually, and according to these studies there is no previous investig-ations dealing
with risk evaluation employing Gumbel’s extreme value theory in Libya further more
the using of a new sets of seismic data file up to 2000 (which presents as recently
compiled preliminary epicentral map of Libya, (Fig.1) rather than 1977.

MAGNITUDE – FREQUENCY RELATIONSHIP

It’s well understand that earthquakes are not independent events, but tend to cluster
in space and time, the knowledge of these trends in time or in space helps in defining
the source regions of future shocks.

Gutenburg and Richter (1954) found that within certain magnitude range a straight
line on semi-log paper fits well most magnitude – frequency distributions.

Typically Log Ne (number of events > Ms) falls off linearily with an increase in M
(earthquake magnitude) and the relationship is usually written as:

\[ \text{Log } N_e (M) = a - bM \quad \text{……………(1)} \]

Where the constant a and b characterised the studied area.

Following, Gutenberg and Richter, the recurrence relationship for the investigated
geographic region is depicted in (Fig.2).
Fig. 1: Seismicity map in and around the Lybian region 1932 – 2000

Fig. 2: Recurrence relationship for the geographic region 21 – 34° N and 8 – 24.5° E (Libya) over the period 1932 – 2000
LogNc(M) = 4.33 − 0.61 * M
The cumulative frequency of earthquakes occurrence is plotted for the period 1932 – 2000 and shown in the same figure, commencing with $M = 2.8$ at interval of 0.4 units, as well, the regression of the given data is revealed in Table (1) as below:

**Table 1: The regression analysis of the given data**

<table>
<thead>
<tr>
<th>Predictor</th>
<th>coefficient</th>
<th>Standard deviation</th>
<th>(T) test</th>
<th>(P) level of mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4.3309</td>
<td>0.1412</td>
<td>30.67</td>
<td>0.00000</td>
</tr>
<tr>
<td>M</td>
<td>-0.60580</td>
<td>0.03062</td>
<td>-19.79</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

$S = 0.1613$  \hspace{1cm} R-Sq = 93.8\%  \hspace{1cm} R-Sq(adj) = 93.5\%$

**Analysis of Variance**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>Ms</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>10.184</td>
<td>10.184</td>
<td>391.46</td>
<td>0.00000</td>
</tr>
<tr>
<td>Error</td>
<td>26</td>
<td>0.676</td>
<td>0.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
<td>10.860</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that the demonstration regression model from $\log N_c (M_s)$ is given with the determination of the magnitude and its recurrences. Having ($R$) correlation coefficient value as $R^2 = 93.8\%$, 6.2\% is not found here which could be referred to the weight of the data used.

However, the regression fit is given by:

$$\log N_c (M_s) = 4.33 - 0.61 M_s$$

$R^2 = 93.8\%$  \hspace{1cm} .................(2)

Where $R^2$ is the correlation coefficient.

From equation (1) we may determine the expected largest magnitude earthquake in the area since 1932 to be :

$$a / b = 7.1$$  \hspace{1cm} .................(3)

this value, however, agrees well with the observed maximum earthquake magnitude of 7.1 $M_s$.

**EXTREME VALUE STATISTICS**

According to Yequlalp and Kuo, 1974; for a relative long sampling period Gumble’s theory of extremes assumes that recurrence of a maximum magnitude earthquake in a given region within such a time interval is a random independent event, and that the behavior of the maximum magnitude earthquake in the future will be similar to that of the past number of observational years. Many workers such as (Yequlalp and Kuo, 1974; and Burton, 1979) give an excellent review of mathematics behind Gumbel’s theory as applied to earthquakes magnitude. For our purposes we make use of the simplified treatment of Lilwall (1976) and Burton (1978) to explain the same concept.
After making the annual extremes of earthquake magnitudes as given in the seismicity file of the investigated area for a sample of consecutive years, the readings are arranged in ascending order as:
M1 < M2 <……….< Mn

Then the ith largest value of magnitude has a probability (Pi) of being an extreme in any ones year and is given by:
Pi (Ms) = i/ (n+1)  ……………(4)

Where Pi is the effective plotting position of the (ith) magnitude observation on the special Gumbel probability paper. It follows that the return period Ti (Ms), which is the mean number of intervals required for a largest value greater than or equal to Ms to be observed, is a monotonically increasing function of (Ms) defined by:
Ti (Ms) = 1 / (1-Pi)  ………….(5)

According to Gumbel (1958) there are three types of extremal asymptotic distributions which corresponding to a specific type of maximum earthquake magnitude behavior. In the first type (type 1) the magnitude series is unlimited and the distribution of the largest value can be defined as:
PI (Ms) = exp[-exp{α (Ms – U2)}]………………(6)

Where α and U are constants.

The second type (type II) introduces a lower limit and the third type (type III) imposes an upper limit (W) where an upper limit exists, P follows the type (III) distribution which given by:
PIII (Ms) = exp [- {W – Ms) / (W – U)}^k]…………………(7)

Where (W) represents the upper bound or limit to the range of extreme values, U is the characteristics extreme value associated with unit time, and k is the curvature parameter allowing for curvature asymptotic to (W) at low annual probabilities or large return periods.

(P) is a probability of non-exceedence of magnitude (Ms).

However, many techniques were found for estimating the extreme distribution parameters which is given by (Gumbel 1958, Yequlalp & Kuo 1974). Amongst these, the least squares and maximum likelihood method were considered and found to give an acceptance and good results. So, the non-linear least square procedure followed by Burton, 1978 to estimate the parameters used by the (Marquardt 1963) Algorithm is employed in the present work.

RESULTS AND DISCUSSION

The gathered earthquake data from 1932 – 2000 which have been occurred in and around Libya (8º – 24.5º E), (21º – 34º N) is used to investigate and determine the recurrence relationship as a magnitude – frequency relationship as well as the estimating distribution of the extreme parameter represented by Gumbel’s theory type (I). So, by such method the probability of magnitude occurrence can be modeled by using the least square method to investigate and determine its statistical extreme value. Having (Gutenberg and Richter 1954) empirical formula, relating the magnitude (M) to the corresponding frequency (N) as given in equation (1) in which the parameters (a and b) give an estimate for the seismic activity in the region under consideration during a
specified period of time. In this investigation and from Fig. (2), both parameters (a and b) were determined using regression analysis, represented by the linear least square solution, as \( a = 4.33 \) and \( b = -0.605 \) With \( R^2 = 93.5\% \) where \( R \) = correlation coefficient. Well, by comparing these results \((a = 4.33, b = -0.605)\) parameters obtained here with those calculated by (Kebeasy, 1980) as \((a = -0.47, b = 0.57)\), it can be said that b parameter is in a close value while the parameter a is entirely differ from the present obtainable value. So, it is opportunity to mention here that either there is a printing mistake in his published paper or might be due to employing the data belongs the only shallow earthquake which then a big difference is obtained.

However, our values revealed here showing more reliability since more events, years even using more sophisticated analysing computer programs which considered in the present study. Well, from equation (1) mentioned previously, we may determine the expected magnitude earthquake in the area under consideration since 1932 to \((a/b = 7.1)\) in which this value agree well with the observed maximum earthquake magnitude of 7.1 Ms. Using the plotting position given by equation (4), Fig.3 illustrate type (I) distribution for the least square technique. On the other hand, Table (2) lists the observed data against the estimated return period (Yr) of earthquakes with magnitude (Ms) for type (I) using such technique. The more or less agreement of the observed return periods with those obtained from type (I) distribution give a positive indication for this estimated procedure.

![Graph showing Gumbel type 1 external distributions for earthquake in the geographic region, 21 – 34° N and 8 – 24.5 ° E (Libya) over the period 1932 – 2000.](image)

\[ F(M) = \exp \left[ - \exp \left( -1.84^* (M - 4.59) \right) \right] \]
Table 2: The observed data versus estimated return periods for Gumbel type (I) distribution using the least square method

<table>
<thead>
<tr>
<th>Magnitude (M)</th>
<th>Observed T(M)(Yr)</th>
<th>Estimated T(M) (Yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>4.5</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>5.0</td>
<td>2.8</td>
<td>2.7</td>
</tr>
<tr>
<td>5.5</td>
<td>6.8</td>
<td>5.9</td>
</tr>
<tr>
<td>6.0</td>
<td>20.3</td>
<td>13.9</td>
</tr>
<tr>
<td>6.5</td>
<td>68.0</td>
<td>34.1</td>
</tr>
<tr>
<td>7.0</td>
<td>100.0</td>
<td>84.9</td>
</tr>
</tbody>
</table>

So, to get and give some idea about the earthquake risk, it is possible to have an idea to the risk and occurrence of a certain magnitude earthquake by application of equation used by Lomnitiz, 1974 as given below:

\[ RD(M) = 1 - [P(M)]^D \] ........................(8)

Where RD is the earthquake risk in the designed period D(Yr).

If we turn now to the Gumbel type (I) distribution and using (\(\alpha\)) symbol, it can be written as

\[ RD(M) = 1 - \exp\left\{-\exp\{\alpha (X - U)\}\right\}^D \] ........................(9)

So, to provide that return period T>D, the earthquake risk can be expressed as:

\[ RT(M) = 1 - \exp\left\{-\exp\{\alpha (X - U)\}\right\}^T \] ........................(10)

By substitution and solving equation (9) and (10), the RD value can get as:

\[ RD = 1 - \exp \left[ \frac{D}{T \ln(1-RT)} \right] \] ........................(11)

Thus, for a given design period (D) it is possible to estimate the risk of occurrence of maximum magnitude earthquakes using the extremes analysis. Table (3), on the other hand, shows risk estimation result using the above mentioned equation (i.e. eq. 11) in conjnction with Gumbel type (I) during certain design periods.

Table 3: Risk estimation during certain design periods for Gumbel type (I) distribution using least square technique

<table>
<thead>
<tr>
<th>Magnitude (M)</th>
<th>Designed Period D(Yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>5.5</td>
<td>0.99</td>
</tr>
<tr>
<td>6.0</td>
<td>0.84</td>
</tr>
<tr>
<td>6.5</td>
<td>0.52</td>
</tr>
<tr>
<td>7.0</td>
<td>0.25</td>
</tr>
<tr>
<td>7.5</td>
<td>0.11</td>
</tr>
</tbody>
</table>

On the basis of Table (3), it can be seen that the extreme value mentioned may show little over estimation of maximum magnitudes in the area under consideration. This result agree with those given by (Weichert and Milne, 1979), (Al-Abbasy and Fahmi, 1985) in which the extreme value theory can give an over estimated maximum magnitudes.
This is true for the whole geographic region under consideration if one bear in his mind, the major maximized magnitude events located outside the bordering of Libya were applied in this statistical study. However, such results may change its behavior if an attempt is made to subdivide this area into smaller source area’s. Well, in these circumstances the obtained results give and reveal its usefulness and reliability about the magnitude levels and earthquake risk in the area.

CONCLUSION

In the present work, an investigation to the recurrence relationship and frequency risk determination is obtained. The more frequencies statistical approach used to analyse the obtainable seismicity data is the least square procedure of Gumbel type (I) extreme distribution for parameter estimation.

Having the analysis of regression, the expected largest magnitude earthquake in the region is determined as \( a/b = 7.1 \) since 1932 which is agree well with the observed maximum earthquake of 7.1 Ms, and could be written as \( 4.33 < M_s < 7.1 \).

By using the Gumbel extreme value theory, Fig. (3) demonstrate the considerable number of observation fall around the optimum line that passing through the distribution observation. The obtainable design period values were statistically calculated with suitable estimation to the risk of occurrence of maximum magnitude in the area under consideration.

In order to go in more detail study, it is recommended that the research should be justifies and extended to involve the application of an aid techniques such as a maximum likelihood method and Gumbel distribution type (III).

REFERENCES

Lilwall, R.C., 1976. Seismicity and seismic hazard in Britain.